Estimation in a dual frame context

A. Arcos, M. Rueda, M. G. Ranalli and D. Molina

November 15, 2024

Contents

1 Introduction

Classic sampling theory usually assumes the existence of one sampling frame containing all finite population units. Then, a probability sample is drawn according to a given sampling design and information collected is used for estimation and inference purposes. But in practice, the assumption that the sampling frame contains all population units is rarely met.

Dual frame sampling approach solves this issue by assuming that two frames are available for sampling and that, overall, they cover the entire target population. The most common situation is the one represented in Figure 1 where the two frames, say frame A and frame B, show a certain degree of overlapping, so it is possible to distinguish three disjoint non-empty domains: domain a, containing units belonging to frame A but not to frame B; domain b, containing units belonging to frame B but not to frame A and domain ab, containing units belonging to both frames.

Then, independent samples s_A and s_B of size n_A and n_B are drawn from frame A and frame B and the information included is suitably combined to provide results.

This vignette shows the way package Frames2 operates and their wide options to work with data coming from a dual frame context.

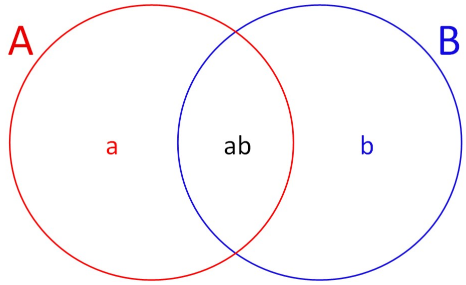

Figure 1: Two frames with overlapping.

2 Data description

To illustrate how functions of the package operate, we will use data sets $DatA$ and $DatB$ (included in the package) as sample data from frame A and frame B, respectively. $DatA$ contains information about $n_A = 105$ individuals selected through a stratified random sampling design from the $N_A = 1735$ individuals composing frame A. Sample sizes by strata are $n_{hA} = (15, 20, 15, 20, 15, 20)$. On the other hand, a simple random without replacement sample of $n_B = 135$ individuals has been selected from the $N_B = 1191$ included in frame B. The size of the overlap domain for this case is $N_{ab} = 601$.

Let see the first three rows of each data set:

```
> library (Frames2)
> data(DatA)
> data(DatB)
> head (DatA, 3)

  Domain   Feed   Clo   Lei     Inc    Tax     M2 Size      ProbA     ProbB
1      a 194.48 38.79 23.66 2452.07 112.90   0.00    0 0.02063274 0.0000000
2      a 250.23 16.92 22.68 2052.37 106.99   0.00    0 0.02063274 0.0000000
3     ab 199.95 24.50 23.24 2138.24 121.16 127.41    2 0.02063274 0.1133501
  Stratum
1       1
2       1
3       1

> head (DatB, 3)

  Domain   Feed   Clo   Lei     Inc    Tax     M2 Size      ProbA     ProbB
1     ba 332.42 38.42 21.12 3109.75 148.07 186.46    3 0.02063274 0.1133501
2      b 222.47 19.94 19.74    0.00   0.00 126.79    2 0.00000000 0.1133501
3      b 215.43 35.13 20.17    0.00   0.00 148.67    3 0.00000000 0.1133501
```

Each data set incorporates information about three main variables: Feeding, Clothing and Leisure. Additionally, there are two auxiliary variables for the units in frame A (Income and Taxes) and another two variables for units in frame B (Metres2 and Size). Corresponding totals for these auxiliary variables are assumed known in the entire frame and they are $T^A_{Inc} = 4300260, T^A_{Tax} = 215577, T^B_{M2} = 176553$ and $T^B_{Size} = 3529$. Finally, a variable indicating the domain each unit belongs to and two variables showing the first order inclusion probabilities for each frame complete the data sets.

Numerical square matrices $PiklA$ and $PiklB$ (also included in the package), with dimensions $n_A = 105$ and $n_B = 135$, are used as probability inclusion matrices. These matrices contains second order inclusion probabilities and first order inclusion probabilities as diagonal elements.

3 Estimation with no auxiliary information

When there is no further information than the one on the variables of interest, one can calculate some estimators, as Hartley (1962, 1974) or Fuller-Burmeister (1972) estimators

```
> data(PiklA)
> data(PiklB)
> yA <- with(DatA, data.frame(Feed, Clo, Lei))
> yB <- with(DatB, data.frame(Feed, Clo, Lei))
> Hartley(yA, yB, PiklA, PiklB, DatA$Domain, DatB$Domain)

Estimation:
             Feed         Clo         Lei
Total 586959.9820 71967.62214 53259.86947
Mean     246.0429    30.16751    22.32556

> FB(yA, yB, PiklA, PiklB, DatA$Domain, DatB$Domain)

Estimation:
             Feed         Clo         Lei
Total 591665.5078 72064.99223 53034.09810
Mean     248.0153    30.20832    22.23092
```

Results show, by default, estimations for the population total and mean for the considered variables. If only first order inclusion probabilities are available, estimates can also be computed

```
> Hartley(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain)

Estimation:
             Feed         Clo        Lei
Total 570867.8042 69473.86532 51284.2727
Mean     247.9484    30.17499    22.2746

> FB(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain)
```

```
Estimation:
             Feed         Clo         Lei
Total 571971.9511 69500.11448 51210.03819
Mean     248.4279    30.18639    22.24236
```

Further information about estimation process (as variance estimations or values of parameters involved in estimation, if any) can be displayed by using function `summary`

```
> summary(Hartley(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain,
+                 DatB$Domain))

Call:
Hartley(ysA = yA, ysB = yB, pi_A = DatA$ProbA, pi_B = DatB$ProbB,
    domains_A = DatA$Domain, domains_B = DatB$Domain)

Estimation:
             Feed         Clo        Lei
Total 570867.8042 69473.86532 51284.2727
Mean     247.9484    30.17499    22.2746

Variance Estimation:
                   Feed          Clo          Lei
Var. Total 9.050344e+08 1.550443e+07 6.977928e+06
Var. Mean  1.707326e+02 2.924874e+00 1.316370e+00

Total Domain Estimations:
                  Feed      Clo      Lei
Total dom. a  263233.1 31476.84 22839.95
Total dom. ab 166651.7 21494.96 15984.64
Total dom. b  164559.2 20451.85 15693.59
Total dom. ba 128704.7 15547.49 11112.38

Mean Domain Estimations:
                 Feed      Clo      Lei
Mean dom. a  251.8133 30.11129 21.84909
Mean dom. ab 241.6468 31.16792 23.17791
Mean dom. b  242.2443 30.10675 23.10221
Mean dom. ba 251.5291 30.38466 21.71707

Parameters:
           Feed       Clo       Lei
theta 0.3787075 0.3358878 0.3362615
```

Results slightly change when a confidence interval is required. In that case, user has to indicate the confidence level desired for the interval through argument `conf_level` (default is `NULL`) and add it to the list of input parameters.

In this case, default output will show 6 rows for each variable, lower and upper boundaries for confidence intervals are displayed together with estimates. So, one can obtain a 95% confidence interval for estimations computed using Hartley and Fuller-Burmeister estimators in this way

```
> Hartley(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain,
+         DatB$Domain, 0.95)

Estimation and  95 % Confidence Intervals:
                   Feed         Clo         Lei
Total       570867.8042 69473.86532 51284.27265
Lower Bound 511904.6588 61756.37677 46106.87729
Upper Bound 629830.9496 77191.35387 56461.66802
Mean           247.9484    30.17499    22.27460
Lower Bound    222.3386    26.82301    20.02587
Upper Bound    273.5582    33.52697    24.52333
```

```
> FB(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain,
+    0.95)

Estimation and  95 % Confidence Intervals:
                   Feed         Clo         Lei
Total       571971.9511 69500.11448 51210.03819
Lower Bound 513045.7170 61802.57411 46036.74627
Upper Bound 630898.1852 77197.65484 56383.33011
Mean           248.4279    30.18639    22.24236
Lower Bound    222.8342    26.84307    19.99541
Upper Bound    274.0217    33.52971    24.48930
```

When, for units included in overlap domaing, first order inclusion probabilites are known for both frames, estimators as the one proposed by Bankier (1986), Kalton and Anderson (1986) can be computed. To do this, numeric vectors `pik_ab_B` and `pik_ba_A` of lengths n_A and n_B should be added as arguments. While `pik_ab_B` represents first order inclusion probabilities according to sampling design in frame B for units belonging to overlap domain selected in sample drawn from frame A, `pik_ba_A` contains first order inclusion probabilities according to sampling design in frame A for units belonging to overlap domain selected in sample drawn from frame B.

```
> BKA(yA, yB, DatA$ProbA, DatB$ProbB, DatA$ProbB, DatB$ProbA,
+     DatA$Domain, DatB$Domain)

Estimation:
             Feed         Clo         Lei
Total 566434.3200 68959.26705 50953.07583
Mean     247.8845    30.17814    22.29822
```

These examples include just a few of the estimators that can be used when no auxiliary information is known. Other estimators, as the pseudo-empirical likelihood estimator (Rao and Wu, 2010) or the dual frame calibration estimator (Ranalli et al., 2014), can be also calculated in this case. In this context, function `Compare` is quite useful, since it returns all possible estimators that can be computed according to the information provided as input

```
> Compare(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain)

$Hartley

Estimation:
             Feed         Clo        Lei
Total 570867.8042 69473.86532 51284.2727
Mean     247.9484    30.17499    22.2746

$FullerBurmeister

Estimation:
             Feed         Clo         Lei
Total 571971.9511 69500.11448 51210.03819
Mean     248.4279    30.18639    22.24236

$PEL

Estimation:
             Feed        Clo         Lei
Total 591956.1900 72391.7894 53396.32780
Mean     247.5017    30.2676    22.32544

$Calibration_DF

Estimation:
             Feed         Clo        Lei
Total 595162.2604 72214.13351 53108.5059
Mean     248.8422    30.19332    22.2051
```

4 Estimation using frame sizes as auxiliary information

Some of the estimators defined for dual frame data, as raking ratio (Skinner, 1991) or pseudo-maximum likelihood estimators (Skinner and Rao, 1996), require the knowledge of frame sizes to provide results. So, frame sizes need to be incorporated to the function through two additional input arguments, `N_A` and `N_B`. There is also a group of estimators, including pseudo-empirical likelihood and calibration estimators, that even being able to provide estimations without the need of auxiliary information, can use frame sizes to improve their precision

```
> SFRR(yA, yB, DatA$ProbA, DatB$ProbB, DatA$ProbB, DatB$ProbB,
+      DatA$Domain, DatB$Domain, N_A = 1735, N_B = 1191)

Estimation:
             Feed        Clo         Lei
Total 596147.5461 72584.5907 53527.39414
Mean     248.1584    30.2148    22.28185
```

```
> CalSF(yA, yB, DatA$ProbA, DatB$ProbB, DatA$ProbB, DatB$ProbB,
+       DatA$Domain, DatB$Domain, N_A = 1735, N_B = 1191)

Estimation:
             Feed         Clo         Lei
Total 595996.8469 72566.50183 53513.52578
Mean     248.1587    30.21495    22.28174
```

Previous estimators need probabilities of inclusion in both frames for the units in the overlap domain to be computed. This condition may be restrictive in some cases. As an alternative, in cases where frame sizes are known but this condition is not met, it is possible to caculate dual frame estimators as pseudo-maximum likelihood, pseudo-empirical likelihood and dual frame calibration estimators

```
> PML(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain,
+     N_A = 1735, N_B = 1191)

Estimation:
             Feed         Clo         Lei
Total 594400.6320 72430.05834 53408.30337
Mean     248.0934    30.23115    22.29178
```

```
> PEL(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain,
+     N_A = 1735, N_B = 1191)

Estimation:
             Feed        Clo         Lei
Total 591956.1900 72391.7894 53396.32780
Mean     247.5017    30.2676    22.32544
```

```
> CalDF(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain,
+       N_A = 1735, N_B = 1191)

Estimation:
             Feed         Clo         Lei
Total 588416.4644 71432.53671 52520.31623
Mean     248.8131    30.20539    22.20832
```

5 Estimation using frame and overlap domain sizes as auxiliary information

In addition to the frame sizes, in some cases, it is possible to know the size of the overlap domain, N_{ab}. Generally, this highly improves the precision of the estimates. Functions implementing pseudo-empirical likelihood and calibration estimators can incorporate overlap domain size to the estimation procedure through parameter `N_ab`, as shown below

```
> PEL(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain,
+     N_A = 1735, N_B = 1191, N_ab = 601)

Estimation:
             Feed         Clo         Lei
Total 575289.2187 70429.95641 51894.32490
Mean     247.4362    30.29245    22.32014

> CalSF(yA, yB, DatA$ProbA, DatB$ProbB, DatA$ProbB, DatB$ProbB,
+       DatA$Domain, DatB$Domain, N_A = 1735, N_B = 1191,
+       N_ab = 601)

Estimation:
            Feed         Clo        Lei
Total 577071.959 70294.89095 51771.9309
Mean     248.203    30.23436    22.2675

> CalDF(yA, yB, DatA$ProbA, DatB$ProbB, DatA$Domain, DatB$Domain,
+     N_A = 1735, N_B = 1191, N_ab = 601)

Estimation:
             Feed        Clo         Lei
Total 578895.6961 70230.1131 51570.55683
Mean     248.9874    30.2065    22.18088
```

6 Estimation using additional variables as auxiliary information

Some of the estimators are defined such that, in addition to frame sizes, they can incorporate auxiliary information about extra variables to the estimation process. This is the case of pseudo-empirical likelihood and calibration estimators. Functions implementing them are also able to manage auxiliary information. To achieve maximum flexibility, these functions are prepared to deal with auxiliary information when it is available only in frame A, only in frame B or in both frames.

For instance, auxiliary information collected from frame A should be incorporated to functions through three arguments: `xsAFrameA` and `xsBFrameA`, numeric vectors, matrices or data frames (depending on the number of auxiliary variables in the frame); and `XA`, a numeric value or vector of length indicating population totals for the auxiliary variables considered in frame A. Similarly, auxiliary information in frame B is incorporated to each function through arguments `xsAFrameB`, `xsBFrameB` and `XB`. If auxiliary information is available in the whole population, it must be indicated through parameters `xsT` and `X`. Let see some examples

```
> PEL(yA, yB, PiklA, PiklB, DatA$Domain, DatB$Domain, N_A = 1735,
+     N_B = 1191, xsAFrameA = DatA$Inc, xsBFrameA = DatB$Inc,
+     XA = 4300260)

Estimation:
             Feed         Clo         Lei
Total 588917.7336 72077.37877 53263.75154
Mean     246.8638    30.21355    22.32722

> CalSF(yA, yB, PiklA, PiklB, DatA$ProbB, DatB$ProbA, DatA$Domain,
+       DatB$Domain, N_A = 1735, N_B = 1191, xsAFrameB = DatA$M2,
+       xsBFrameB = DatB$M2, XB = 176553)

Estimation:
            Feed         Clo         Lei
Total 581539.671 70735.99535 52208.48996
Mean     247.159    30.06336    22.18902

> CalDF(yA, yB, PiklA, PiklB, DatA$Domain, DatB$Domain, N_A = 1735,
+       N_B = 1191, xsAFrameA = DatA$Inc, xsBFrameA = DatB$Inc,
+       xsAFrameB = DatA$M2, xsBFrameB = DatB$M2, XA = 4300260,
+       XB = 176553)

Estimation:
             Feed         Clo         Lei
Total 585185.4497 71194.61148 52346.43878
Mean     247.8075    30.14866    22.16705
```

While pseudo-empirical likelihood estimator has been computed considering only auxiliary information in frame A, single frame calibration estimator has been calculated considering auxiliary information in frame B. For the dual frame calibration estimator, auxiliary information in both frames has been taken into account.

References

[1] Arcos, A., Molina, D., Rueda, M. and Ranalli, M. G. (2015). *Frames2: A Package for Estimation in Dual Frame Surveys.* The R Journal, 7(1), 52 - 72.

[2] Bankier, M.D. (1986). *Estimators Based on Several Stratified Samples With Applications to Multiple Frame Surveys.* Journal of the American Statistical Association, Vol. 81, 1074 - 1079.

[3] Fuller, W.A. and Burmeister, L.F. (1972). *Estimation for Samples Selected From Two Overlapping Frames* ASA Proceedings of the Social Statistics Sections, 245 - 249.

[4] Hartley, H.O. (1962). *Multiple Frame Surveys.* Proceedings of the American Statistical Association, Social Statistics Sections, 203 - 206.

[5] Hartley, H.O. (1974). *Multiple frame methodology and selected applications.* Sankhya C., Vol. 36, 99 - 118.

[6] Kalton, G. and Anderson, D.W. (1986) *Sampling Rare Populations.* Journal of the Royal Statistical Society, Ser. A, Vol. 149, 65 - 82.

[7] Ranalli, M.G., Arcos, A., Rueda, M. and Teodoro, A. (2014). *Calibration estimators in dual frames surveys.* arXiv:1312.0761 [stat.ME].

[8] Rao, J. N. K. and Wu, C. (2010) *Pseudo Empirical Likelihood Inference for Multiple Frame Surveys.* Journal of the American Statistical Association, Vol. 105, 1494 - 1503.

[9] Skinner, C.J. (1991). *On the Efficiency of Raking Ratio Estimation for Multiple Frame Surveys.* Journal of the American Statistical Association, Vol. 86, 779 - 784.

[10] Skinner, C. J. and Rao, J. N. K. (1996). *Estimation in Dual Frame Surveys With Complex Designs.* Journal of the American Statistical Association, Vol. 91, 349 - 356.